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RAINFALL DEPTH-DURATION
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THIS PAPER

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SYNOPSIS

It has long been established that the hydrologic cycle abides by rigorous physical laws. However, the translation of its cyclic-progress to quantitative terms has yet to be accomplished. Rationally, it can be assumed that although the physical appearance of a natural phenomena may be progressively changing as it continues through a sequence of events, there will be maintained a constant equality between the activating forces and resultant effects.

It is the intention of this paper to present the results of studies which indicated that the complex nature of most short-time precipitations, regardless of patterns, depths, durations and frequencies, are reducible to simple "common-denominator" exponential equations, and that the incremental rainfall periods are usually interdependently proportional to the total storm.

INTRODUCTION

Precipitation contains a number of dimensional characteristics of which two prime properties are depth and duration. Both independent variables are essential for a collection of recordings to be statistically determinate; however, it is an exception rather than a rule to find data wherein these quantities are recorded adequately for positive evaluation.

An inconstant characteristic of rainstorms is the "pattern" or chronologic arrangement of the sub-intensities. This actually seems to be haphazard in sequence because an infinite combination of peaks and lows are possible. There are intense storms of short durations and gentle rains of long durations. The peaks might occur early, as in the convective-type storms, or perhaps appear in the later stages, as in frontal induced rains.

An examination of the recordings of two maximum storms of equal depths reveals that for one the peak intensities occurred in the second five-minute period and for the other in the fifth five-minute period. In other words, the sequence and magnitude of the storm increments are inconstant even for similar type precipitations.

Excellent papers[2,3,4] have been written on the probable occurrences of sub-intensities within any rainfall period. Very interesting and practicable results have been obtained therefrom in their application to the solution of

1. Project Engr., Porter, Urquhart, McCreary & O'Brien, Cons. Engrs., Newark, N. J.
2. E. R. Breihan, Civ. Engrg., May 1940. p. 303.
3. D. I. Blumenstock, Techn. Bull. No. 698, U.S.D.A.
4. G. R. Williams, Hydrology, Engineering Hydraulics, p. 275.

precipitation pattern distribution.[5] Therefore, it is considered unnecessary to discuss this phase except to emphasize the erratic performance of precipitation.

Studies of short-time maximum precipitation records[6] reveal further inconstancies. Comparisons of events show that a wide range of depths can occur within any given duration. For example, at Galveston, Texas, on October 5-6, 1910, and Sandy Hook, New Jersey, on August 10, 1931, recordings were 6.28 and 1.40 inches respectively, yet both storm durations were identical.

Furthermore, to add to the complexities, any specific combination of depth and duration can be further categorized, with relation to other known occurrences at a given station, by the determination of its frequency, or the time when that particular combination will be reached or exceeded again. This probability value is not constant with reference to the magnitude of a storm, as it may vary with the location of the observation point. Its prime importance lies in establishing maximum limits for design capacities of structures above which it would not be feasible, or at least uneconomical to provide for, unless major hazards are involved.

Obviously, there are innumerable meteorological factors affecting the rainstorm structure. Attempting to correlate dimensionally all the vacillating properties, wherein the possible maximum latitudes of the simplest constituent part is enormous, is a prodigious task. However, by converting the element values into a form of non-dimensional ratios, a method of comparison becomes available whereby the effect of difference in elemental magnitudes is reduced to a minimum. This method of analysis will be described and discussed in subsequent paragraphs of this paper.

Relationships of Precipitation

I. Basic Data:

In conjunction with the drainage design for several airbases in French Morocco, North Africa, hydrologic studies were made in an effort to determine the magnitudes of the 2-year and 10-year frequency design storms. It was necessary to obtain a thorough understanding of the climatic, meteorological and hydrological conditions of the area so that the calculations for the sizes of culverts, ditches, ponding basins and other drainage facilities would be of sufficient capacity to adequately discharge the excess rainfall, or runoff from the large apron pavements and complex bay systems between the runway and taxiways.

As is often the case, the paucity of records and inadequacy of those available precluded their use in establishing positive frequency-intensity-duration relationships.

There were approximately fifteen government agricultural experimental stations near the locations of the proposed construction where weather observation posts were maintained of which only one utilized an automatic rainfall recording device. Usually the records consisted of 12-hour total rainfall depth measurements for the determination of monthly and seasonal trends,

5. "Surface Runoff Determination from Rainfall Without Using Coefficients" by W. W. Horner and S. W. Jens, Trans. ASCE, Vol. 107, 1942, pp-1039-1075.
6. "Rainfall Intensity—Frequency Data," by D. L. Yarnell, Misc. Publication No. 204, U.S.D.A., 1935, pp. 9-23.

and it was rare that a notation was made as to the type of storm or specific duration of the precipitation periods. It was known, however, that most rains were the result of convective or orographic activity, as Morocco is in the fringe areas of the Atlantic Polar fronts and is protected by mountain ranges from the effects of Continental Polar air.

Under close examination, it was apparent that the data were incomplete and inconsistent when comparisons were made with records from other stations which were under the influence of the same storm. The dissimilarity of the measurements for stations of relatively close proximity pointed to a laxity in observational practices rather than the meteorological trend associated with precipitation depth-area relationship.

It was obvious that in order to perform an adequate analysis, it would be necessary to correct and synthesize parts of the erratic records so that they could be evaluated statistically and a method conceived whereby the 12-hour depths could be converted into one-hour intensities. On the other hand, if this conversion could be accomplished and a relationship ascertained, in order to test the validity of the conclusions, it would be necessary to compare the results with other actual local short-time precipitation records, none of which were available.

One of the results from the studies of rainfall data in the United States, reported in a paper[7] by Gail A. Hathaway, M. ASCE, was that there existed a similarity between storm intensities of one-hour durations and those of lesser periods, regardless of the frequency of occurrence of the storms or the geographical location of the precipitation stations. Presumably, this structural uniformity indicates that the mechanical processes of the storm cell are not affected by the location or configuration of terrain over which the event occurs. However, the prevailing climatic conditions at the storm location will directly influence the magnitude of the activating forces and, consequently, indirectly determine the quantity, pattern, and duration of the resultant precipitation. In other words, the mechanics are identical in the development of all rainfalls and the mathematics of the process can be applied to all storms, providing modifications are made for climatic differences.

In the absence of local records, the collection of maximum intensity, short-time precipitations tabulated in the paper[6] by David L. Yarnell was used as the basic data for determining the relationships between the incremental periods and total storm depths, as being broadly representative of intense, short-time precipitation. They contained a varied pattern of peak occurrence, and the total depths and durations ranged from a low of 0.51 inches for 15 minutes to a high of 9.03 inches for 12 hours. The varied characteristics of the storms are shown by the typical mass curves in Figure No. 1.

Most of these storms can be generally classified as being convective, with the peak intensities occurring in the early periods. Geographically, they represent events covering areas from the delta regions of the Gulf of Mexico to the high levels of the Rocky Mountains, and from the Hudson Valley to the Great Lakes. The majority of the rains fell in the summer seasons and is proportioned at approximately 75 per cent for the months of June to September with 25 per cent for the months of March to June.

Consequently, it was believed that any conclusions resulting from the studies of these dissimilar storms would significantly demonstrate the existence of any mechanical independence within the storm and thus could be realistically applicable to other remote localities.

7. "Design of Drainage Facilities, Military Airfields," by Gail A. Hathaway, Trans. ASCE, Vol. 110, 1945.

II. Methodology:

The sixty storms used in these studies were chosen because of the large differences in their depths, durations, and patterns. Conversely, the characteristic was a prime difficulty in arriving at a convenient method of analysis. Examination of the observed precipitation records indicated an apparent lack of uniformity between the sequences of period-depths in the individual rain and no semblance of a proportion in the period-depths for similar events.

However, the "standard supply curves" prepared by Mr. Hathaway[7] indicate there is a certain reliability in the appearance of a given average depth of rainfall for any specific duration, although it is generally conceded that the average intensities will be greatly exceeded within the sub-periods of the storm.

It is reasonable to assume that there must be an interrelationship between the sub-intensities and the total depths, since there is uniformity in the appearance of the total depths themselves, as shown by the aforementioned supply curves.

To facilitate plotting and subsequent evaluation, it was necessary to reorganize the rainfall data in an attempt to eliminate the wide variances in the component parts namely, depth, duration and pattern, and to provide a common working datum. This was accomplished by establishing the total storm depths to equal unity, and then computing what portion of this value was embodied within the intermediate maximum-periods. These calculations produced a series of incremental depths, with the maximum-periods expressed as a percentage of the total rainfall. Instead of using the actual observed sub-periods, the maximum-periods were utilized as a device by which the sequence of precipitation as a fluctuating variable would be excluded.

Storm durations were subdivided into proportional parts in a similar manner. The time-period, wherein the previously computed maximum-precipitation percentage had occurred, was reduced to a percental portion of the total storm time. Thus both variables could be plotted with similar non-dimensional coordinates regardless of chronological length or measured depth.

The only remaining variable, pattern, was eliminated as a deterrent to direct storm comparison by arranging the maximum-precipitation periods in ascending order of magnitude, regardless of where they chronologically occurred in the actual storm. That is, the maximum 5-minute depth was listed first; the maximum 10-minute depth, second; the maximum 15-minute depth, third; until the total storm was thus tabulated.

This arrangement precluded any effect the erratic performances of "pattern" might have on the analysis. The maximum 5-minutes might have been the observed third 5-minute period, and the maximum 20-minutes might have been composed of the first, second, third and fourth 5-minute periods. Regardless of the sequence of the maximum-periods, a definite portion of the total depth had been accumulated within a specific sum of incremental parts of the total storm and in each case it would be an absolute percentage thereof. This is shown in Table No. 1 which is a partial listing of the storms studied.

The coordinates obtained from this tabulation were plotted on semi-logarithmic paper with the depths as the ordinates and the durations as the abscissae. The traces of the storms, when plotted in this manner, progressed very uniformly. The mass trace occupied a relatively narrow band on the graph as shown in Figure No. 2.

This indicated the existence of a constant ratio between succeeding parts of a storm and a certain dependence of one period on the other pertaining to its position and magnitude. It also showed that, regardless of when the peak intensities occur in a storm, whether early or late, the relationship between the period-depth and the total depth is fairly constant. Conversely, the ratio of the total storm depth and duration will remain fairly constant with the maximum sub-period depths and durations.

All storm plottings maintained a remarkable degree of similarity with one exception, the storm of June 21, 1933, at Burlington, Vermont. This particular event was obviously a peculiarity, as more than 75 per cent of the total rain fell in less than 10 per cent of the total storm duration. At this point the rainfall rate diminished and the maximum-period precipitation trace entered the mass plottings before an additional 10 per cent of the rainfall had accrued. Thereafter the trace continued uniformly and in conformance with the other recordings. It was believed that this erratic storm behavior would have had little influence upon the balance of the picture. Therefore it was disregarded as a weighing factor in the analysis.

To facilitate determination of a curve definition, upon completion of the delineation of storm tabulations, upper and lower limits were superimposed upon the mass plottings. Averaging these outer boundaries, mean curves representive of three groups of twenty storms each were drawn as shown in Figure No. 3.

This shape indicated a form of exponential relationship expressed mathematically as

$$y = ax^n$$

with "y," the rainfall depth expressed as a percentage of the total depth; "x," the duration expressed as a percentage of the total duration; "a," a constant; and "n," the exponent of the percental duration. This will, when plotted on logarithmic paper, assume a linear form.

All three mean curves were plotted in this manner and numerical values were determined for the constant and exponent of the parameter. These ranged between 3.06 and 3.54 for the constant and 0.810 to 0.856 for the exponent. At a point between 30 and 40 per cent of total storm time, the slope of the average line lessened and the values of the constant and exponent changed. They then varied between 11.70 to 19.50 and 0.355 to 0.480 respectively. A mean precipitation curve was plotted from these as shown in Figure No. 4.

III. Comparisons:

In order to confirm the empirical equation derived from the mean-curve graph, comparisons were made with other rainstorm recordings. Inasmuch as all the records, used as the basis for these studies, denoted the most intense rainfall recorded at each of the sixty stations, it was believed that a firmer validation would result from comparison with other types of precipitations.

Thirty storms were used as the base for the comparisons and these events generally can be classified as frontal rains. Further comparisons were made with a number of other most-intense rainfalls, as shown in Figure No. 5, and a number of separate recordings for various stations within an area covered by one particular storm, as shown in Figure No. 6. The storms were divided into ten parts of equal duration and, with the use of the "common denominator" equation, the percental maximum-depths for each sub-period were computed

and a series of sub-period maximum-depth-duration values derived.

The synthetic incremental depths were obtained from these quantities by subtracting the maximum 5-minute value from the maximum 10-minute value; the 10-minute from the 15-minute, et cetera. These were then arranged as shown in Table No. 2 so that the maximum periods of both the synthetic and actual storms were chronologically juxtaposed.

The observed precipitations were plotted as mass curves and the synthetic traces superimposed. For all practical purposes the plottings were identical with the maximum deviation being approximately 0.40 inches or about 13 per cent less than the actual accumulated total from the storm record at Lexington, Kentucky, as shown on Figure No. 5.

CONCLUSIONS

It is believed that these studies corroborate the probabilities established by the Corps of Engineers' Standard Supply Curves[7] and are indicative of a parallel probability in the appearance of accumulative maximum sub-intensities in the individual rainstorm. Also, they indicate that the haphazard appearance of rainfall is primarily the result of the innumerable possible combinations and magnitudes of the actuating forces. For any given combination of values the resultant traces will be in conformity with rigid limiting laws.

Although the basis for the empirical equations herein described are short-time maximum-intensity recordings, there are indications that they may be compatible with longer duration, lesser intensity rainfalls. Studies are being conducted at present toward this end and also toward determining its adaptability as a means for deriving one-hour intensities from 6—or 12-hour measurements, such as those collected at many agricultural stations. An additional program is concerned in evaluating its applicability to area—depth—duration relationships.

Runoff analyses of complex drainage basins have been made utilizing synthetic design storms, arranged in logical patterns and based upon previously determined frequency, depth and duration values. Comparisons of the quantities obtained by this method with actual measured amounts for the particular areas indicate that a practical solution to runoff problems may be available with a considerable reduction of the time-consuming research necessary at the present.

More research is required in determining the effect of the activating forces of precipitation upon the pattern of rainstorms. The combined efforts and cooperation of the hydrologist and meteorologist will undoubtedly be the means to the solution of this problem. The interrelationship of the sciences dictates the need of a close association between the respective groups so that each acquire a broader understanding of the mechanics of storm phenomena.

ACKNOWLEDGMENTS

Table No. 1

PERIOD DEPTH-DURATION PERCENTAGE OF TOTAL STORM DEPTH-DURATION

STATION		Duration of Period - Minutes														Total Storm
		5	10	15	20	25	30	35	40	45	50	60	80	100	120	
Abilene, Texas	Max.Pre.	.66	1.08	1.48	1.77	2.03	2.24	2.37	2.41							2.41 ins.
	% Prec.	27	45	61	73	84	88	97	100							40 mins.
	% Dur.	13	25	38	50	63	75	88	100							
Alpena, Mich.	Max.Pre.	.53	.94	1.04												1.04 ins.
	% Prec.	51	90	100												15 mins.
	% Dur.	33	67	100												
Baker, Oreg.	Max.Pre.	.16	.30	.46	.58	.70	.78									0.78 ins.
	% Prec.	21	38	59	74	90	100									30 mins.
	% Dur.	17	34	50	67	85	100									
Boise, Idaho	Max.Pre.	.17	.33	.35	.37	.44	.54	.62	.69	.78	.81	.95				0.95 ins.
	% Prec.	18	35	37	39	46	57	65	68	82	85	100				60 mins.
	% Dur.	9	17	25	33	42	50	58	67	75	83	100				
Broken Arrow, Okla.	Max.Pre.	.28	.54	.80	1.01	1.18	1.35	1.52	1.69	1.78	1.90	2.23	2.86	3.10		3.10 ins.
	% Prec.	9	17	26	33	38	44	49	55	57	61	72	92	100		100 mins.
	% Dur.	5	10	15	20	25	30	35	40	45	50	60	80	100		
Brownsville, Texas	Max.Pre.	.64	1.28	1.69	2.08	2.46	2.94	3.26	3.57	3.83	4.19	4.75	5.63	5.76	5.81	5.81 ins.
	% Prec.	11	22	29	35	42	50	55	60	65	71	80	95	97	100	120 mins.
	% Dur.	3	6	8	11	14	17	19	22	25	28	33	44	56	100	
Burlington, Vt.	Max.Pre.	.46	.85	.87	.89	.89	.90	.92	.95	.97	.99	1.03	1.08	1.09		1.09 ins.
	% Prec.	42	78	80	82	82	83	85	87	91	94	97	99	100		100 mins.
	% Dur.	5	10	15	20	25	30	35	40	45	50	60	80	100		
Charleston, S.C.	Max.Pre.	.53	1.02	1.48	1.87	2.16	2.36	2.57	2.77	3.04	3.51	4.08	5.28	6.12	6.62	6.62 ins.
	% Prec.	8	15	22	28	33	36	39	42	46	53	62	80	93	100	120 mins.
	% Dur.	3	6	8	11	14	17	19	22	25	28	33	44	56	100	

Table No. 2

DEVELOPMENT OF SYNTHETIC RAINFALS

EVANSVILLE, IND.
August 10, 1908   TOTAL STORMS

Depth: 2.50 ins.   Duration: 60 mins.

Duration of Period	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Max. Prec. For Period	.48	.82	1.15	1.36	1.56	1.74	1.89	2.13	2.33	2.45		2.50				
Obs. Precipitation	.12	.18	.20	.21	.34	.48	.33	.15	.24	.20	.02	.02				

SYNTHESIS:

	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Percent of Duration	8.3	16.7	25.0	33.3	41.7	50.0	58.3	66.6	75.0	83.3		100				
Percent of Precipitation	18.9	33.8	47.3	59.8	69.5	75.1	79.6	84.6	88.9	92.3		100				
Syn. Max. Precipitation	.47	.85	1.18	1.49	1.74	1.88	1.99	2.12	2.22	2.31		2.50				
Syn. Precipitation	.09	.13	.25	.31	.38	.47	.33	.11	.13	.10	.10	.09				

LEXINGTON, KY.
July 3, 1931   TOTAL STORMS

Depth: 3.26 ins.   Duration: 80 mins.

Duration of Period	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Max. Prec. For Period	.66	1.13	1.54	1.83	2.10	2.34	2.63	2.85	2.96	3.03		3.15				3.26
Obs. Precipitation	.06	.29	.47	.66	.41	.27	.24	.29	.22	.11	.07	.06	.03	.03	.03	.02

SYNTHESIS:

	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Percent of Duration	6.3	12.5	18.8	25.0	31.3	37.5	43.7	50.0	56.3	62.5		75.0				100
Percent of Precipitation	15.0	26.6	37.1	47.4	56.8	66.0	71.3	75.0	78.8	82.4		88.8				100
Syn. Max. Precipitation	.49	.87	1.21	1.55	1.85	2.15	2.32	2.44	2.57	2.69		2.88				3.26
Syn. Precipitation	.09	.34	.38	.49	.34	.30	.30	.17	.12	.13	.12	.09	.10	.10	.09	.09

TYPICAL MASS CURVES
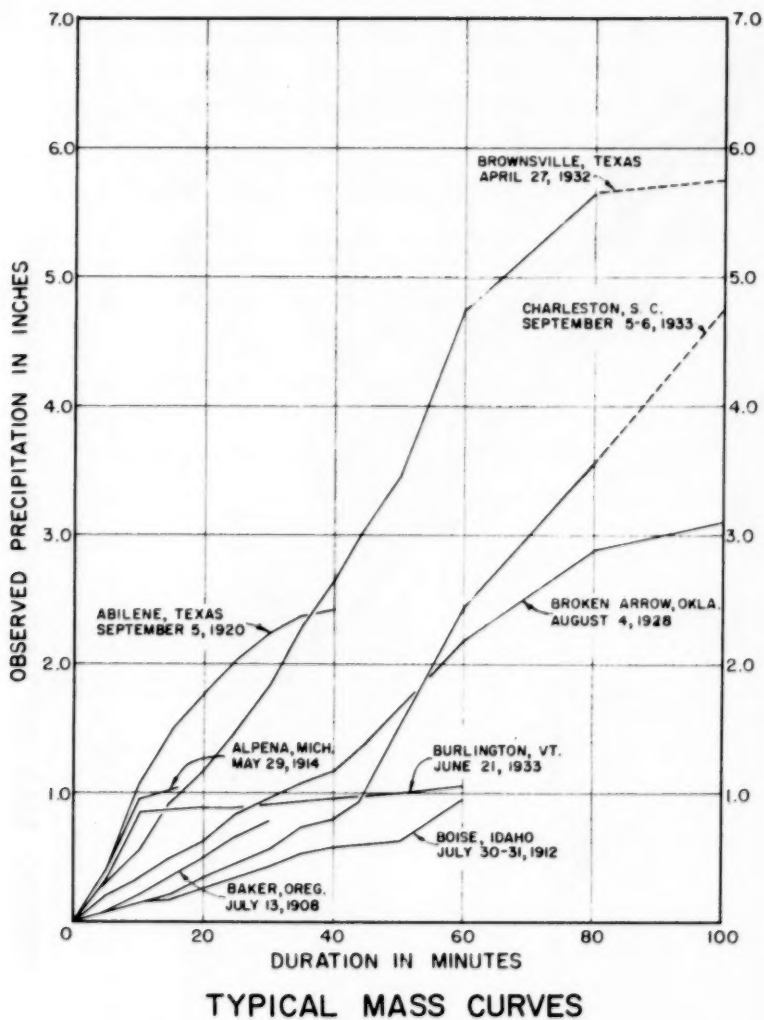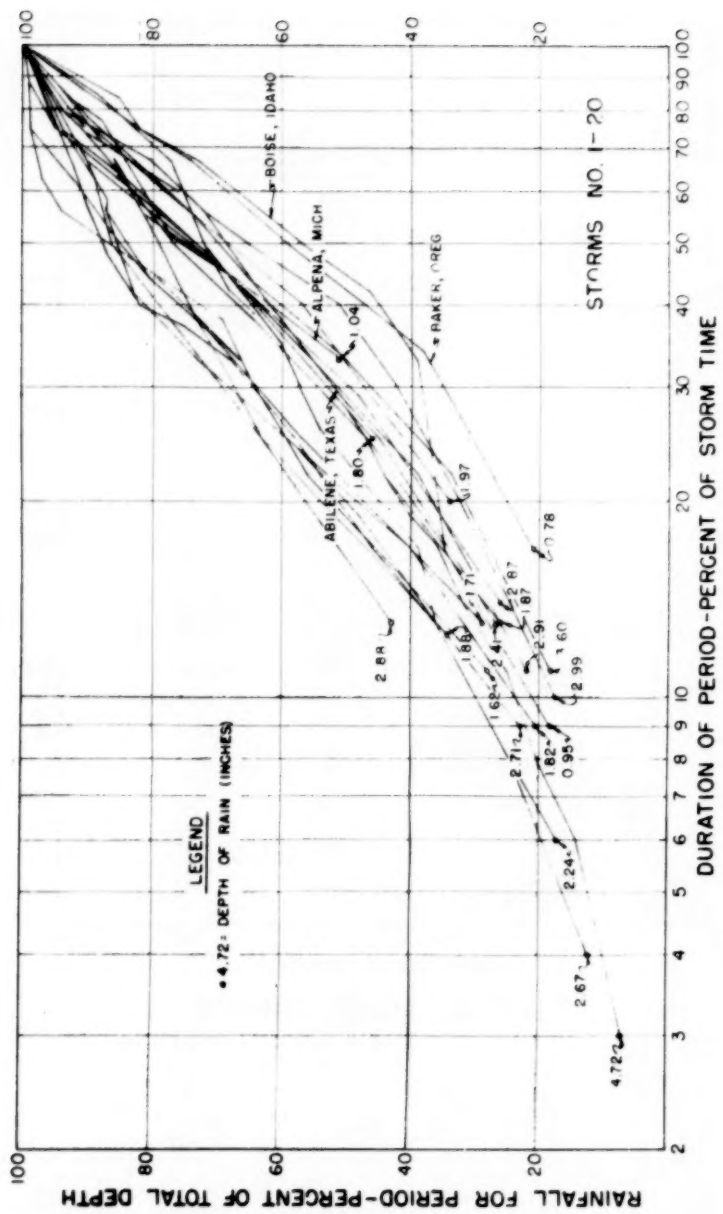
PERIOD MAX. DEPTH-DURATION RELATIONSHIP

FIGURE NO 2

840-10

FIGURE NO. 3

MAX. DEPTH-DURATION LIMITS

STORMS NO. 1-20

DURATION OF PERIOD-PERCENT OF STORM TIME

RAINFALL FOR PERIOD-PERCENT OF TOTAL DEPTH

UPPER LIMITS

AVERAGE STORM

LOWER LIMITS

840-11

MEAN MAX. PRECIPITATION CURVE

DURATION OF PERIOD-PERCENT OF STORM TIME

RAINFALL FOR PERIOD-PERCENT OF TOTAL DEPTH

STORMS NO. 1-60

$Y = 14.90 X^{0.413}$
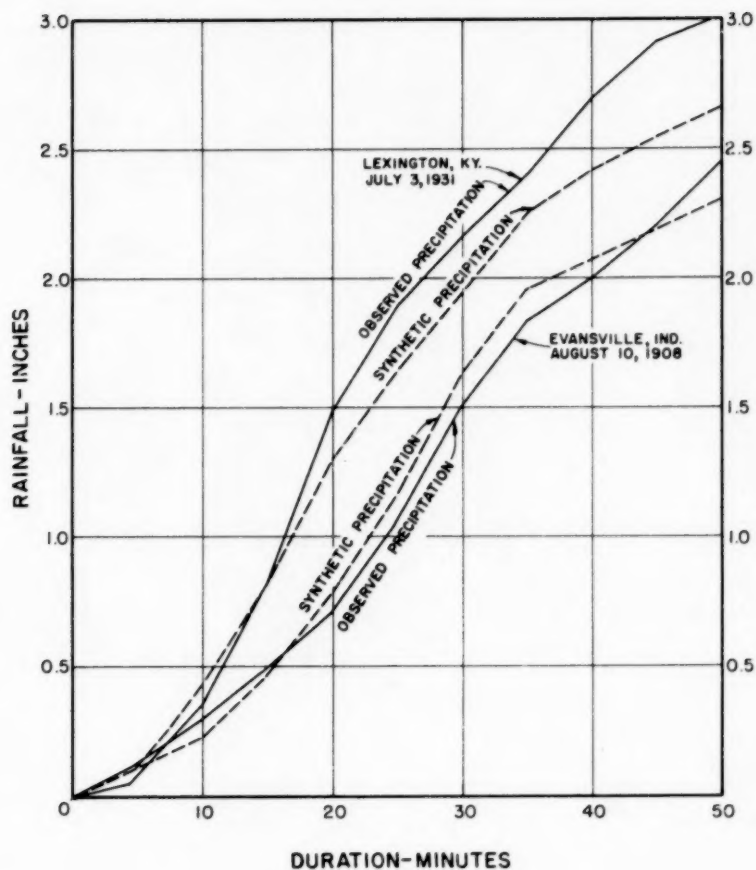
$Y = 3.25 X^{0.831}$

FIGURE NO. 4

840-12
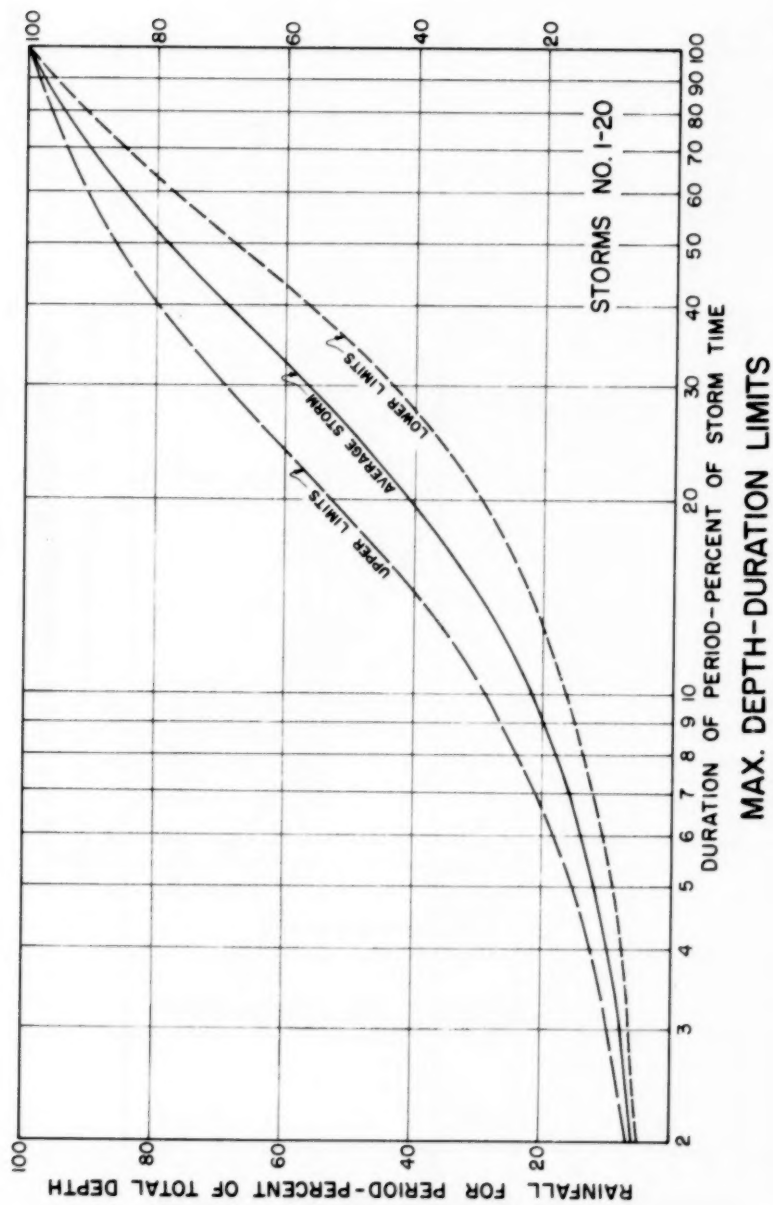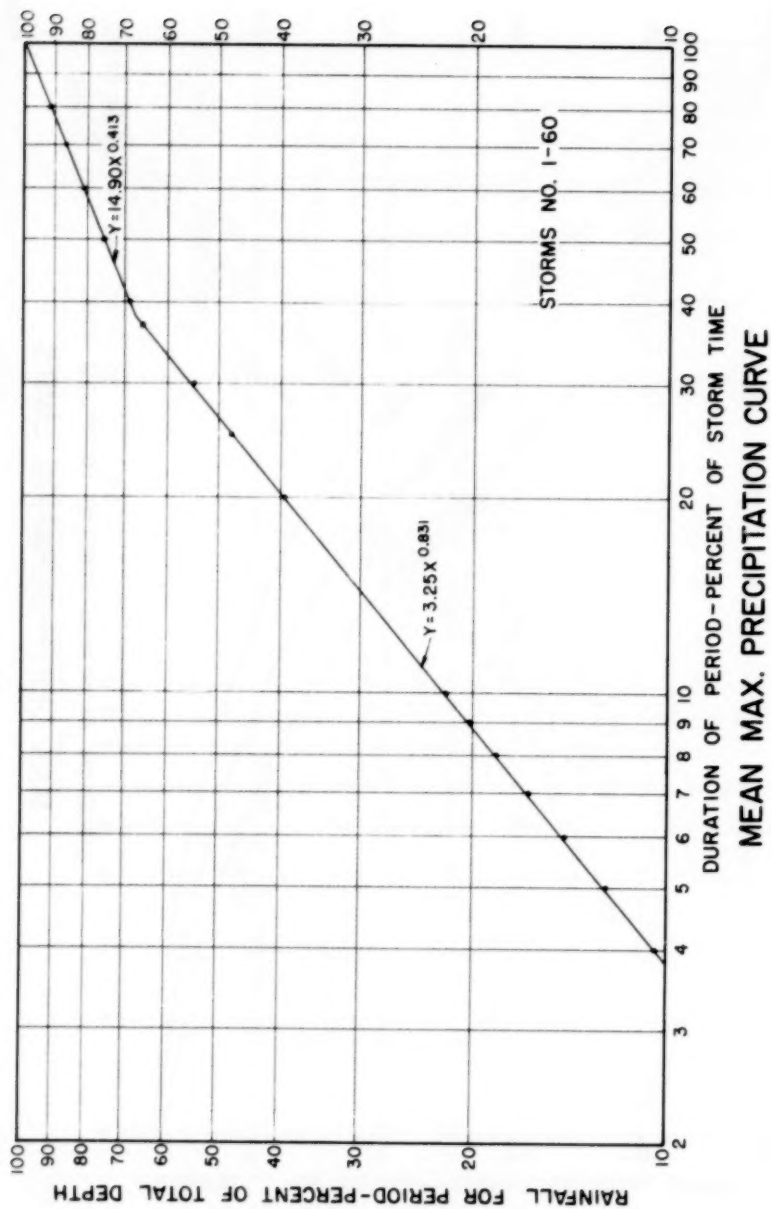
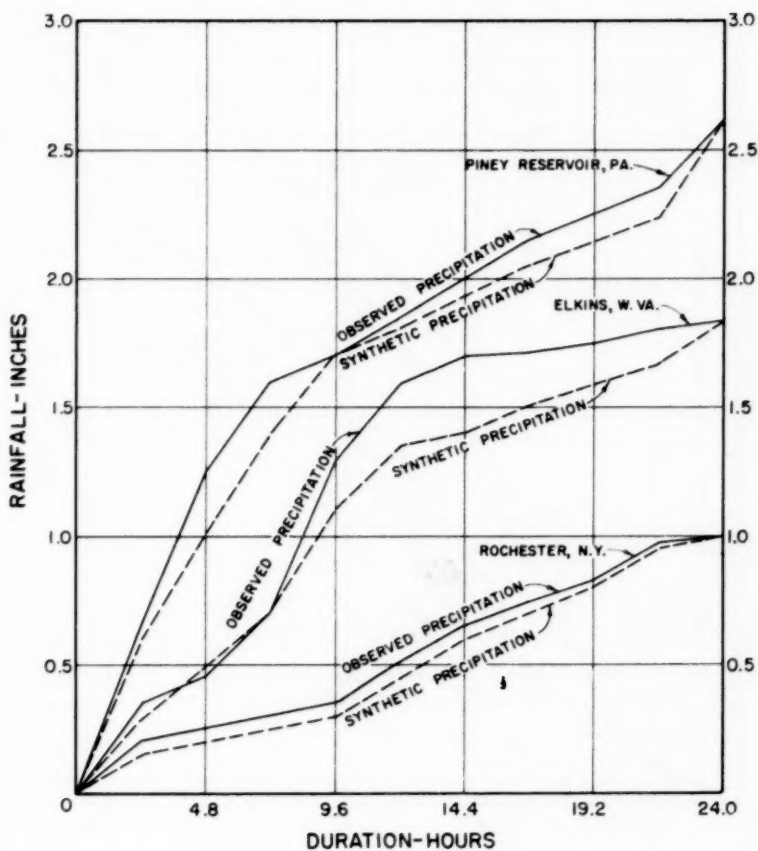SYNTHESIS OF RAINFALLS

FIGURE NO. 5

SYNTHESIS OF LONG DURATION STORM
MARCH, 1936 DURATION APPROX. 3 DAYS
(24 HOUR PERIOD)

FIGURE NO. 6